An Efficient Robust Algorithm for the Surface-Potential Calculation of Independent DG MOSFET

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Abstract—Although the recently proposed single-implicit-equation-based input voltage equations (IVEs) for the independent double-gate (IDG) MOSFET promise faster computation time than the earlier proposed coupled-equations-based IVEs, it is not clear how those equations could be solved inside a circuit simulator as the conventional Newton–Raphson (NR)-based root finding method will not always converge due to the presence of discontinuity at the G-zero point (GZP) and nonremovable singularities in the trigonometric IVE. In this paper, we propose a unique algorithm to solve those IVEs, which combines the Ridders algorithm with the NR-based technique in order to provide assured convergence for any bias conditions. Studying the IDG MOSFET operation carefully, we apply an optimized initial guess to the NR component and a minimized solution space to the Ridders component in order to achieve rapid convergence, which is very important for circuit simulation. To reduce the computation budget further, we propose a new closed-form solution of the IVEs in the near vicinity of the GZP. The proposed algorithm is tested with different device parameters in the extended range of bias conditions and successfully implemented in a commercial circuit simulator through its Verilog-A interface.

Index Terms—Circuit simulation, compact modeling, double-gate MOSFET, input voltage equations (IVEs).

I. INTRODUCTION

THE independent double-gate (IDG) MOSFET has received considerable attention in the recent years, owing to its ability to modulate the threshold voltage and the transconductance dynamically. A fast and accurate solution of the input voltage equations (IVEs) is the most fundamental step toward developing surface-potential-based compact models for such transistors. Previous techniques [1]–[4] used for solving the 1-D Poisson equation (PE) rigorously for long-channel IDG MOSFETs result in IVEs that involve multiple intercoupled implicit equations, which are computationally expensive for circuit simulation. Recently, we have proposed a different rigorous technique for solving the same PE by which one can obtain single-implicit-equation-based IVEs [5] (there are four independent implicit equations; however, for a given bias condition, we need to solve only one of them). By using the optimization routine available in a commercial computational software program [6], we show that the single-implicit-equation-based IVEs converge five times faster than the coupled-implicit-equation-based IVEs. However, it is difficult to make such optimization routines converge, and they are also not available in standard “C” or “Verilog-A” libraries.

Finding the solution of the IVEs for the IDG MOSFET is a nontrivial exercise due to the following reasons. In comparison to the bulk and symmetric double-gate (SDG) MOSFETs, the complexities in the IVEs of the IDG MOSFET are manifolds. Here, the IVEs can take two different forms, i.e., trigonometric (when the external-bias-dependent coupling factor \( G \leq 0 \)) or hyperbolic (\( G > 0 \)), and thus, the IVEs are discontinuous at \( G = 0 \) [G-zero point (GZP)]. In addition, the trigonometric IVE has several nonremovable singularities due to the presence of \( \cot() \) and \( \log[\sin()] \) terms. As a result, the Newton–Raphson (NR)-based method either does not converge (leads to an imaginary solution) or very slowly converge when the root of the IVEs lies close to the discontinuity/singularity points. Moreover, the higher order derivatives of the IVEs are extremely cumbersome, and thus, it is impractical to constitute a higher order NR method for faster convergence. In order to achieve assured convergence, one needs to use a computationally expensive bisection method. Furthermore, it can be seen that the analytical approximations proposed for bulk [7] and SDG MOSFETs [8] are not applicable to the IVEs of the IDG MOSFET.

In this paper, we propose a unique algorithm to solve the IVEs [5], which combines the Ridders algorithm [9] with the NR-based technique in order to provide assured convergence with a low computational budget for any bias conditions. The Ridders algorithm is a bounded root finding technique, which offers assured convergence at a much faster rate than the regular bisection routine. Studying the IDG MOSFET operation carefully, we apply an optimized initial guess to the NR component and a minimized solution space to the Ridders component in order to achieve rapid convergence, which is very important for circuit simulation. To reduce the computation budget further, using the concept of the GZP, we propose a new closed-form solution of the IVEs in the very near vicinity of the GZP. The proposed algorithm is implemented in a Verilog-A-based
circuit simulator [10], tested and verified against numerical simulations [11], [12].

It is worth noting that coupled-implicit-equation-based IVEs also have similar discontinuity and singularity points. However, as it is extremely difficult to constitute an efficient bounded root finding technique for the multidimensional solution space [13], those IVEs appear to be more theoretical in nature than useful for the practical implementation in a circuit simulator. A similar argument is also applicable for a recent work [14] that uses complex variable and multiple coupled equations.

II. DEVELOPMENT OF THE ALGORITHM

Conventions used in this paper are follows: \( C_{ox1(2)} \) is the oxide capacitance per unit area of the first (second) gate defined as \( \epsilon_{si}/t_{ox1(2)} \), \( C_{si} \) is the silicon body capacitance per unit area defined as \( \epsilon_{si}/t_{si} \), \( \epsilon_{si} \) and \( \epsilon_{ox} \) are the permittivities, and \( t_{si} \) and \( t_{ox} \) are the thicknesses of silicon and SiO\(_2\), respectively. \( q \) is the elementary charge, \( \beta \) is the inverse thermal voltage, \( n_i \) is the intrinsic carrier density, \( V \) is the electron quasi-Fermi potential (channel potential), \( \psi(2)_i Si/SiO_2 \) is the interface potential at first (second) gate, and \( V_{gs1(2)} \) is the effective front (back)-gate voltage, i.e., \( V_{gs1(2)} = V_{gs1(2)} \) applied \( \Delta \phi_{1(2)} \), where \( \Delta \phi_{1(2)} \) is the work function difference at the respective gates.

We explain the algorithm for case \( V_{gs1} \geq V_{gs2} \), and it could be extended in similar lines for case \( V_{gs1} < V_{gs2} \). The hyperbolic \( f_{hyp1}(\psi_1) = 0 \) and trigonometric \( f_{trig1}(\psi_1) = 0 \) equations for which we are seeking the solutions are given below:

\[
f_{hyp1} = -\frac{B e^{-\beta V}}{G_1} \sinh \theta_1 - e^{-\beta(V_{gs2} - V)} G_1 \frac{\sinh \theta_1}{\cosh \theta_1} = 0
\]

\[
f_{trig1} = \sqrt{G_1^2 - G_1 \sinh \theta_1} - \frac{e^{-\beta(V_{gs2} - V)}}{B} = 0
\]

where

\[
G_1 = \frac{C_{ox1}^2 V_{gs1} - \psi_1^2}{C_{si}^2} - B e^{-\beta(V_{gs1} - V)}
\]

\[
\theta_1 = G_1 \frac{h_{si}}{2} + \sinh^{-1} \left( \frac{\sqrt{G_1}}{\sqrt{e^{-\beta(V_{gs2} - V)}}} \right)
\]

with \( B = (2q n_i)/\beta \epsilon_{si} \) and \( G_1^* = -G_1 \). \( \theta_1^* \) could be obtained by replacing \( G_1 \) with \( G_1^* \) and \( \sinh^{-1} \) with \( \sin^{-1} \) in (4). These equations are obtained by restructuring the IVEs proposed in [5] for better convergence.

A. \( \psi_{exp1} \) Calculation

As explained in [5], the surface-potential computation should start with the \( \psi_{exp1} \) calculation, which is the solution of the 2\( G_1 = 0 \) equation. As the IVEs are discontinuous at the GZP, \( \psi_{exp1} \) needs to be calculated with very high accuracy. Although a closed-form solution of \( \psi_{exp1} \) is available in terms of the Lambert W function and can be implemented in a simulator as discussed in [15], such an expression has following limitations:

1) The exponential term inside the argument of the W function overflows for high values of \( (V_{gs1} - V) \) (for example, beyond 36 V at room temperature for a long double-precision data-type implementation) and thus becomes uncomputable for large bias values, which might appear during circuit simulation [16].

2) The implementation of the W function in special C libraries or in mathematical packages are generic in nature. For example, in the GNU Scientific Library (GSL) [17], the W function is computed using Halley’s method with some optimized initial guess (obtained from the polynomial approximation of the W function). It is possible to reduce the computation time of \( \psi_{exp1} \) if we try to solve equation \( G_1 = 0 \) directly using a physics-based initial guess. In this paper, we calculate \( \psi_{exp1} \) by transforming equation \( G_1 = 0 \) into the following form:

\[
f_{psi_{exp1}}(\psi_1) = V_{gs1} - \psi_1 - \frac{\epsilon_{si} \sqrt{B e^{-\beta(\psi_1 - V)}}}{C_{ox1}} = 0.
\]

We then solve this equation by Halley’s method using the initial guess \( V_{gs1} \) for \( V_{gs1} \leq 0 \) and \( \min((2/\beta) \ln(V_{gs1} C_{ox1}/\epsilon_{si}\sqrt{B}) + V_{gs1}) \) for other values with the exit condition \( |f_{psi_{exp1}}| < 10^{-17} \) or when the absolute difference between \( \psi_{exp1} \) obtained from two successive iterations becomes less than 10\(^{-6}\) (where \( \delta \) is the machine precision and, for a long double data type, \( \delta \) is equal to 2\(^{-52} \approx 2.2 \times 10^{-16} \)). In most of the cases, the Halley loop is seen to converge within two to three iterations. We present the comparison of the performance of our proposed implementation to that of the Lambert-function-based approach in Section III.

We then use \( \psi_{exp1} \) to calculate the value of the critical gate voltage \( V_{gs2_{crit}} \) (as it appears in [5]) in order to choose between trigonometric or hyperbolic IVE.

B. Explicit Solution

For a given bias point, \( \psi_1 \) is determined, as depicted by the algorithm in Fig. 1. Here, we first check for the cases where \( \psi_1 \) can be approximated to an explicit form in order to reduce the computation budget. As explained in [5], at deep weak inversion, the difference between \( \psi_{exp1} \) and \( \psi_{gsp1} \) becomes extremely small, and then, the surface potential could be approximated by the following expression of \( \psi_{1wi} \) by neglecting the inversion charge completely:

\[
\psi_{1wi} = \frac{V_{gs1}}{1 + \frac{C_{ox1} C_{si}}{C_{ox1} C_{si} + C_{ox2} C_{si}}} + \frac{V_{gs2}}{C_{ox2} C_{si} + C_{ox2} C_{si} + 1}.
\]

Another explicit formulation of the surface potential is possible when \( V_{gs2} \) is very close to \( V_{gs2_{crit}} \) (i.e., the solution is very close to the \( \psi_{exp1} \) value). Here, using the concept of the GZP, a generic explicit formulation for the surface potential could be obtained by Taylor series expansion with respect to \( V_{gs2} \) around the GZP as given below:

\[
\psi_1 = \psi_{exp1} + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n \psi_1}{\partial V_{gs2}^n} |_{V_{gs2} = V_{gs2_{crit}}} (V_{gs2} - V_{gs2_{crit}})^n.
\]

Unfortunately such expression cannot be used in the circuit simulator as it is very difficult to obtain the analytical
Algorithm for finding roots of the IDG-MOSFET IVEs. ERT, ERH are adjustable parameters chosen to be $10^{-17}$ and $10^{-13}$ respectively in this work.

Fig. 1. Algorithm for finding roots of the IDG-MOSFET IVEs. ERT, ERH are adjustable parameters chosen to be $10^{-17}$ and $10^{-13}$ respectively in this work.

Expressions for higher order derivatives $\frac{\partial^n \psi_1}{\partial V_{gs2}^n}$ at the GZP. However, if we neglect all the higher order terms in (7), then in the close neighborhood of the GZP, $\psi_1$ could be approximated as $\psi_1 \approx \psi_{1r} = \psi_{exp1} + (\partial \psi_1/\partial V_{gs2})|_{\psi_1=\psi_{exp1}} (V_{gs2} - V_{gs2crit})$, where the expression for $\partial \psi_1/\partial V_{gs2}|_{\psi_1=\psi_{exp1}}$ is obtained from the IVEs (either from the trigonometric or hyperbolic one) and given in (8), shown at the bottom of the next page.
The bias ranges for which such solutions could be used in the algorithm is heuristically chosen such that the continuity in the surface potential, the drain current, and the transconductances is well satisfied. If conditions for using those explicit solutions are not satisfied, the IVEs are numerically solved. The explicit solution also helps us avoid the computation of IVEs (1) and (2) around the GZP, which is numerically unstable, since \( \cot(\theta) \rightarrow \infty \) as \( G \rightarrow 0 \).

**C. Solution of the Hyperbolic IVE**

In order to solve \( f_{\text{hyp1}} \), we first use the NR algorithm with the optimized initial guess and then switch to the Ridders algorithm if NR fails to converge. In order to use the Ridders algorithm, we first need to calculate the solution space (i.e., maximum and minimum possible values of \( \psi_1 \)). In the hyperbolic mode, \( \psi_1 < \psi_{\text{gap1}} \) [5], and thus, the maximum bound for \( f_{\text{hyp1}} = 0 \) is \( \psi_{\text{gap1}} \). The lower bound could be obtained in following manner. When \( G_1 \) becomes large, [5, eq. (11)] could be approximated as

\[
C_{\text{ox2}} \left( \frac{2}{\beta} \ln \left( \frac{B_{\text{c}} ^{-\beta V}}{G_1} \right) + \theta_1 - \ln 2 \right) + V_{gs2} = 0.
\]

The solution of (9) gives the lower bound of \( \psi_1 \) in \( f_{\text{hyp1}} \). However, this equation is difficult to solve. Since \( \sinh^{-1}(x) > \ln(2x) \) for all positive and finite \( x \) values and \( \ln(2x) \) is a curvilinear asymptote of \( \sinh^{-1}(x) \), by replacing term \( \sinh^{-1}\left(\sqrt{G_1/\sqrt{BE^{(\psi_1-V)}}} - \ln 2 \right) \) with \( \ln\left(\sqrt{G_1/\sqrt{BE^{(\psi_1-V)}}}\right) \) in the expression for \( \theta_1 \) in (9), we arrive at the following simpler function \( f_{\text{lt_hyp1}} \), the solution of which, i.e., \( \psi_{\text{limit}} \), provides the lower bound of \( \psi_1 \):

\[
f_{\text{lt_hyp1}} \equiv B e^{-\beta(\psi_1)} - \frac{C_{\text{ox2}}}{\psi_{\text{gap1}}^2} (V_{gs1} - \psi_1)^2 \]

\[
+ \frac{C_{\text{ox2}}^2}{\psi_{\text{gap1}}^2 \left( \frac{\psi_{\text{gap1}}}{\psi_{\text{gap1}}^2} + 1 \right)^2} (V_{gs2} - \psi_1)^2. \tag{10}
\]

Using the technique discussed in [8], an approximate analytical solution of (10) is obtained, as shown in Fig. 1. To solve \( f_{\text{hyp1}} \) when \( V_{gs1} - V \leq 0.4 \) or \( V_{gs2} < 0.01 \), the initial guess \( \psi_{\text{guess}} \) for NR is chosen to be \( \min(\psi_{\text{gap1}} - \delta, \psi_{\text{min}}) \) since the device will be either in weak inversion or close to the GZP. For other biases, \( \psi_{\text{guess}} \) is taken to be \( \psi_{\text{limit}} \). The derivative of \( f_{\text{hyp1}} \) is given in (11). As one can see, the expression of \( f_{\text{hyp1}} \) is pretty complicated, and thus, it is impractical to constitute the higher order NR method using higher order derivatives. The NR loop successfully converges if \(|f_{\text{hyp1}}| < 10^{-13}\) or the change in \( \psi_1 \) between two successive iterations, i.e., \( |\psi_{\text{new}} - \psi_{\text{old}}| \), is less than \( \delta \).

As mentioned earlier, \( f_{\text{hyp1}} \) is discontinuous at the GZP, and thus, for many practical biases, the NR loop might not converge if, at any stage, the iterative solution becomes higher than \( \psi_{\text{gap1}} \). For such cases, we have to switch to the Ridders algorithm. We implement the Ridders algorithm as it appears in [18] with \( \psi_{\text{max}} \) and \( \psi_{\text{min}} \) representing quantities \( x_h \) and \( x_l \), respectively. A slight modification is made to eliminate the cases where the proposed implementation might fail. When the S-parameter given in [18] goes to zero or \( |(F(\psi_{\text{max}} + \psi_{\text{min}})/|S| < 10)\), where \( F \) is the function to be solved by the Ridders algorithm, then the bisection algorithm is used as backup to assure certain convergence. This situation is however very less probable, as shown in the results presented Section III.

As shown in Fig. 1, a flag \( \text{flag_ridders} \) gets set whenever \( \psi_{\text{new}} \) exceeds \( \psi_{\text{gap1}} \). It also gets set when the relative change in the solution between iterations \(|\psi_{\text{new}} - \psi_{\text{old}}| \) is less than \( 10^{-4} \), which is an empirical check for slow NR convergence or oscillations in convergence. When this flag gets set, the Ridders algorithm gets executed after breaking out of the NR loop. For the Ridders method, the ideal bounds for the solution spaces are \( \psi_{\text{gap1}} \) and \( \psi_{\text{limit}} \). However, in the hyperbolic mode, \( \psi_{\text{limit}} \) can be very far from \( \psi_{\text{gap1}} \), and thus, the Ridders algorithm might slowly converge. The interim solution \( \psi_{\text{old}} \) obtained in the NR loop is used to reduce the bounds of the solution space. As shown in Fig. 1, the sign of function \( f_{\text{lt_hyp1}} \) at the interim solution \( \psi = \psi_{\text{old}} \) compared with its sign at \( \psi = \psi_{\text{gap1}} \). If the sign is negative, then the lower bound can be changed from \( \psi_{\text{limit}} \) to \( \psi_{\text{old}} \), else the upper bound can be changed from

\[
\frac{\partial \psi_1}{\partial V_{gs2}} \bigg|_{\psi=\psi_{\text{gap1}}} = 12 \alpha \beta C_{\text{ox2}}^2 C_{\text{si}}^2 \left( \alpha \beta t_{\text{si}} + 2 \right)^3 \]

\[
\left( B^2 \beta t_{\text{si}}^3 (16 \alpha t_{\text{si}} + 10 C_{\text{ox2}} t_{\text{si}} + \alpha \beta C_{\text{ox2}} t_{\text{si}}^2 + 2 \alpha \beta t_{\text{si}} + 24) \right) \frac{e^{3\beta(\psi_{\text{gap1}})}}{e^{3\beta(\psi_{\text{gap1}})}} \]

\[
+ \frac{16 BC_{\text{ox2}} (\alpha \beta t_{\text{si}} + 6) - \beta t_{\text{si}}^2 (5 \alpha \beta t_{\text{si}} + 9)}{e^{3\beta(\psi_{\text{gap1}})}} \frac{B (8 \beta t_{\text{si}}^2 (2 \alpha \beta t_{\text{si}} + 3))}{e^{3\beta(\psi_{\text{gap1}})}} \]

\[
- 96 \alpha \omega_1 (\psi_{\text{gap1}} - C_{\text{ox2}} + 2 t_{\text{si}}) \frac{2 B^2 \beta t_{\text{si}} (\alpha \beta t_{\text{si}} + 10)}{e^{2\beta(\psi_{\text{gap1}})}} - \frac{2 B^2 \beta t_{\text{si}} (\alpha \beta t_{\text{si}} + 10)}{e^{2\beta(\psi_{\text{gap1}})}}
\]

\[
\text{where} \quad \alpha_1 = \sqrt{B e^{-\beta(V_\psi_{\text{exp}})}} \quad \text{and} \quad \omega_1 = C_{\text{ox1}}^2 (\psi_{\text{gap1}} - V_{gs1}). \tag{8}
\]
ψ_{g_{zp1}} to ψ_{old}. The search space, hence, is now limited to either ψ_{old} to ψ_{lim} or ψ_{old} to ψ_{g_{zp1}} instead of ψ_{lim} to ψ_{g_{zp1}}. In this way, even in cases where NR does not converge, we can use its interim solution ψ_{old} to reduce the search space, which remarkably improves the efficiency of the Ridders algorithm.

\[
f'_{hyp1} = -\sqrt{\frac{B e^{-\beta V}}{G_1} \cosh(\theta_1) \kappa_1 + \frac{B e^{-\beta V} \sinh(\theta_1) \gamma_1}{2 G_1}} \]

\[
= \frac{\beta}{2} \left( \frac{e_{ox} \cosh(\theta_1) \gamma_1}{2 C_{ox2} G_1} - \frac{e_{ox} \cosh(\theta_1) (\cosh^2(\theta_1) - 1) \kappa_1}{C_{ox2}} \right) + \frac{\beta}{e} \left( V_{gs} + \frac{G_1 \cosh(\theta_1)}{2} \right)
\]

where

\[
\gamma_1 = \frac{\partial G_1}{\partial \psi_1} = \frac{2 C_{ox1}(\psi_1 - V_{gs1})}{e_{si}} - \beta B e^{-\beta(V - \psi_1)}
\]

\[
\kappa_1 = \frac{\partial \theta_1}{\partial \psi_1} = \frac{\beta}{4 \sqrt{G_1}} - \frac{\beta e_{ox} \gamma_1}{2 B e^{\beta V} e^{-\beta V_{gs1}}} + \frac{B \gamma_1}{2 G_1 e^{\beta V} e^{-\beta V_{gs1}}} + \frac{B \gamma_1}{2 G_1 e^{\beta V} e^{-\beta V_{gs1}}} + \frac{B \gamma_1}{2 G_1 e^{\beta V} e^{-\beta V_{gs1}}}
\]

\[
= -\frac{\beta e_{ox} \gamma_1}{4 \sqrt{G_1}} - \frac{2 \sqrt{G_1} B e^{-\beta V_{gs1}}}{B e^{-\beta V_{gs1}} + 1}
\]

(11)

**D. Solution of the Trigonometric IVE**

To solve \( f_{trig1} = 0 \), we use only the Ridders algorithm as the solution space is much smaller (five to ten times) than the hyperbolic case, where the Ridders method is found to show similar or even better performance than the NR or hybrid NR–Ridders method. Moreover, the Ridders algorithm does not require the derivative of the function to be solved. As the derivatives of the IVEs are pretty complicated, the implementation of the Ridders algorithm is less error prone than the NR routine. The lower bound for the solution space is \( \psi_{g_{zp1}} \) as in the trigonometric mode \( \psi_1 > \psi_{g_{zp1}} \) [5]. From the expression of \( f_{trig1} \), one can deduce that the upper bound of the solution space \( \psi_{lim} \) is determined by the solution of equation \( f_{trig1} = \pi - \theta_1 = 0 \), which also denotes the first nonremovable singularity point of \( f_{trig1} \) and thus needs to be very accurately evaluated. Unlike \( f_{hyp1} \), the solution space of \( f_{trig1} \) is bounded between a discontinuity point and a singularity point, which gives another argument to use pure Ridders method for root finding.

A straightforward NR-based method is found to be not efficient for solving \( f_{trig1} \), as during iteration, \( \sqrt{G_1} \) could become imaginary, and thus, we use the hybrid NR–Ridders method (similar to the hyperbolic mode). To solve \( f_{trig1} \) by the Ridders method, we need to find the upper bound of \( \psi_{lim} \) (the lower bound is given by \( \psi_{g_{zp1}} \)). Now, as the maximum possible value of \( \sqrt{G_1} \) could be \( 2 \sqrt{\pi} / (\beta e_{si}) \), the solution of equation \( \pi - (\beta \sqrt{G_1} e_{si}) = 0 \), denoted by \( \psi_{1X} \), provides the upper bound of \( \psi_{lim} \). This equation for computing \( \psi_{1X} \) can be transformed into following form, which can eliminate the discontinuities at boundaries and has better convergence:

\[
f_{1X} = V - \psi + \ln \left( \frac{4 \sqrt{\sin(\psi - V_{gs1})^2 + C_{ox1}(\psi - V_{gs1})^2}}{\beta} \right) - \ln \left( \frac{B}{\beta} \right). \tag{12}\]

\( f_{1X} \) can be solved using Halley’s method, as shown in Fig. 1. Although the solution of \( f_{1_{trig1}} \) gives the ideal upper bound for the surface potential, it can be further reduced to optimize the solution space of \( f_{trig1} \). In order to do so, we use the fact that \( \psi_1 \leq V_{gs1} \); however, \( \psi_{lim} \) could be greater than \( V_{gs1} \). As shown in Fig. 1, if \( V_{gs1} \) is less than \( \psi_{lim} \), then the upper bound for \( \psi_1 \) is simply \( V_{gs1} \), and thus, the computation of \( \psi_{lim} \) can be avoided. This check is done even before the computation of \( \psi_{1X} \) by comparing the sign of function \( f_{1_{trig1}} \) at \( \psi = V_{gs1} \) with that of \( \psi = \psi_{g_{zp1}} \). This optimization in the upper bound of \( \psi_1 \) is found to provide significant improvement in the overall computation time, as we not only avoid the overhead \( \psi_{lim} \) computation, but we also reduce the solution space for the solution in \( f_{trig1} = 0 \). It will be shown in Section III that the overhead computation time is quite small compared with the total computational-time Ridders loop in (2), and thus, the reduction in its solution space will improve the overall performance significantly. However, this check has a numerical limitation. It can be performed only when \( V_{gs1} - V < 708 / \beta \), as beyond this point, the exponential term in the expression of \( f_{1_{trig1}} \) overflows (similar to the \( \psi_{g_{zp1}} \) calculation). Hence, for cases \( V_{gs1} - V > 708 / \beta \), the aforementioned bound optimization is performed after the computation of \( \psi_{lim} \). Once the optimized upper bound for \( \psi_1 \) is obtained, \( f_{trig1} \) is solved using the Ridders method, as explained in Fig. 1.

III. RESULTS AND DISCUSSION

We validate the accuracy of our algorithm against the numerical solution of the PE obtained from the COMSOL 4.0 multiphysics software [12]. Fig. 2 shows the good agreement between results obtained from the proposed algorithm and the
COMSOL data for the calculation of the surface potential and its derivative for both cases where \( V_{gs1} \) is greater and less than \( V_{gs2} \). The analytical expressions for the derivatives are not shown due to space constraints. In Fig. 3, we have shown how the proposed algorithm handles the discontinuity at the GZP. In this figure, we have plotted \( g_{m2}/I_D \) around \( V_{gs2crit} \) for \( V_{gs1} = 1 \) V. (Circles) COMSOL data. (Solid lines) Solution by our algorithm. The device parameters are \( t_{ox1} = 1 \) nm, \( t_{ox2} = 2 \) nm, and \( t_{si} = 10 \) nm. Here, we used the tied gate configuration with \( \Delta \phi_1 = 0.56 \) and \( \Delta \phi_2 = -0.56 \).

Finally, in Fig. 8, we show the histogram of the iteration needed (excluding the \( \psi_{1limit} \) calculation) to compute \( \psi_1 \) for dev1 and dev2, respectively. Bars in black represent samples in the hyperbolic mode, and bars in gray represent that in the trigonometric mode. It can be noticed that convergence is always achieved within ten iterations. It also shows that the probability of using the Ridders algorithm in the hyperbolic mode is very small and the bisection method has never been used. The explicit solution also gets used (although the probability is very small) both in the trigonometric and hyperbolic modes, as indicated by the presence of samples with zero iteration.

Fig. 5 shows the histogram of the total computation time (including the \( \psi_{1limit} \) calculation) taken by the samples for nonzero iterations. The histogram is generated by binning the total samples into bins of 0.1-ms width based on the magnitude of their total computation time. Hence, for example, a sample with a 0.32-ms computational time falls into the 0.4-ms bin, and a sample with a 1.15-ms computation time falls into the 1.2-ms bin. It is shown that, in both the devices, the peak in trigonometric samples lags behind the peak in hyperbolic samples by about 9%–15%. Fig. 7 shows the histogram of the percentage of the total computation time taken by the \( \psi_{1limit} \) calculation for samples in the trigonometric mode for dev1 and dev2, respectively. On an average, it is shown that the \( \psi_{1limit} \) computation, i.e., the overhead computation needed in the trigonometric mode, takes about 10% of the total computation time in both the devices. From Fig. 5, it is shown that the trigonometric and hyperbolic modes peak at the same iteration number. From these observations, it could be concluded that the computation of the trigonometric IVE is slower than that of the hyperbolic IVE because of the iterative \( \psi_{1limit} \) calculation in the trigonometric mode.

COMSOL data for the calculation of the surface potential and its derivative for both cases where \( V_{gs1} \) is greater and less than \( V_{gs2} \). The analytical expressions for the derivatives are not shown due to space constraints. In Fig. 3, we have shown how the proposed algorithm handles the discontinuity at the GZP. In this figure, we have plotted \( g_{m2}/I_D \), where \( g_{m2} \) is the second gate transconductance and \( I_D \) is the drain current, against \( V_{gs2} \), while the source end is changing from hyperbolic to trigonometric mode (by varying \( V_{gs2} \) around \( V_{gs2crit} \) for a step size of 1 nV). Both the characteristics appear to be continuous and match very well with the COMSOL data. It should be noted that, in regime \( |V_{gs2} - V_{gs2crit}| < 10^{-6} \), we use the explicit formulation, but beyond this limit, we solve the IVEs numerically.

In Fig. 4, we examine the robustness of the proposed algorithm (and the \( \psi_{gsp} \) calculation) against extremely high bias voltages, which might appear during circuit simulation. As the overflow and the underflow of math functions under extreme biases are properly covered in the algorithm, it gives an accurate result for any bias conditions.

Due to the presence of several independent variables in the IVEs, it is very difficult to analyze the computation efficiency of the proposed algorithm. We use the following statistical technique in order to have a fair estimate of the efficiency. For two different devices dev1 and dev2 (having \( t_{ox1} = 2 \) nm, \( t_{ox2} = 3 \) nm, and \( t_{si} = 30 \) nm, and \( t_{ox1} = 1 \) nm, \( t_{ox2} = 1 \) nm, and \( t_{si} = 10 \) nm, respectively), the efficiency of the algorithm is demonstrated in terms of the required number of iterations and computation time by executing it for 1 million random bias points where \( V_{gs1} \) and \( V_{gs2} \) are varying between 0 and 3 V for a channel voltage of 0 V.

Fig. 6 shows the histogram of the total computation time (including the \( \psi_{1limit} \) calculation) taken by the samples for nonzero iterations. The histogram is generated by binning the total samples into bins of 0.1-ms width based on the magnitude of their total computation time. Hence, for example, a sample with a 0.32-ms computational time falls into the 0.4-ms bin, and a sample with a 1.15-ms computation time falls into the 1.2-ms bin. It is shown that, in both the devices, the peak in trigonometric samples lags behind the peak in hyperbolic samples by about 9%–15%. Fig. 7 shows the histogram of the percentage of the total computation time taken by the \( \psi_{1limit} \) calculation for samples in the trigonometric mode for dev1 and dev2, respectively. On an average, it is shown that the \( \psi_{1limit} \) computation, i.e., the overhead computation needed in the trigonometric mode, takes about 10% of the total computation time in both the devices. From Fig. 5, it is shown that the trigonometric and hyperbolic modes peak at the same iteration number. From these observations, it could be concluded that the computation of the trigonometric IVE is slower than that of the hyperbolic IVE because of the iterative \( \psi_{1limit} \) calculation in the trigonometric mode.
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Fig. 5. Histogram of the iterations needed to compute $\psi_1$ for 1 million samples. Samples in (black bars) hyperbolic mode and (gray bars) trigonometric mode. The Ridders algorithm in the hyperbolic mode is used for samples in the bar represented by “R.” For dev1, 771,011 samples are in the trigonometric mode, whereas for dev2, 739,998 samples are in the trigonometric mode.

Fig. 6. Histogram of the total computation time for the samples with nonzero iterations. Samples in (black bars) hyperbolic mode and (gray bars) trigonometric mode.

Fig. 7. Histogram of the percentage of the total computation time taken by the $\psi_1$ limit calculation for samples in the trigonometric mode.

In a very recent work [19], effort has been also put to implement those IVEs [5] in a Verilog-A-based circuit simulator with some mathematical conditioning. As the detailed algorithm simulated using the circuit simulator and verified with the mixed-mode simulation of a technology computer-aided design (TCAD) simulator [11].
Fig. 8. Histogram depicting the ratio of computational times between the proposed approach to the Lambert-function-based approach for $\psi_{gpp1}$ using a million random bias points using a bin width of 0.1 for binning the ratios.

Fig. 9. Simulated characteristics of NAND and AND gates using (line) the circuit simulator and (symbol) the mixed-mode TCAD simulator, keeping the second input at $V_{DD} = 1$ V. Here, we use the same $W/L$ ratio ($W$ is the width, and $L$ is the length of the transistor) for both p- and n-FETs. Yet, the logic transition at $V_{DD}/2$ is achieved by applying suitable second gate voltages (0.8 and 0 V for all p- and n-FETs, respectively). Device parameters used are $t_{ox1} = 1$ nm, $t_{ox2} = 2$ nm, and $t_{si} = 10$ nm.

(initial guess, exit condition, etc.) is not presented, we are not able to compare the proposed algorithm with their work.

IV. CONCLUSION

We have proposed a robust yet efficient root finding algorithm for the IVEs of an IDG MOS transistor without any compromise in accuracy. The proposed algorithm uses a unique approach of root finding by combining the Ridders algorithm with the NR method in order to provide assured convergence in the presence of discontinuity and singularity in the IVEs. Physics-based optimized input guess, minimized solution space, and regional explicit solution have been used to make the computation faster. The algorithm has been successfully implemented in a commercial circuit simulator and verified against numerical simulations.

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